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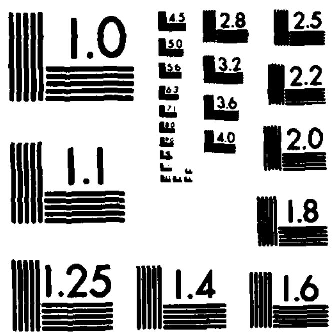

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Chapel Hill, North Carolina

A Trivariate Version of 'Lévy's Equivalence'

by

Gordon Simons

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It is shown that the trivariate stochastic processes $\{(M_t, W_t, M_t, \Theta_t, t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions when $W = \{W_t, t\} \geq 0$ is a Wiener process, M_t is the maximum value attained by W over the time interval $[0,t]$, Θ_t is the time the maximum value is attained, L_t is the local time of W at level zero and time t, and T_t is the last time W is zero in the time interval $[0,t]$. A straightforward proof, based on "Tanaka's formula", establishes this result by deriving an almost sure version of the equivalence.

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A Trivariate Version of 'Lévy's Equivalence'

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Summary

It is shown that the trivariate stochastic processes $\{(M_t - W_t, M_t, \Theta_t), t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions when: $W = \{W_t, t \geq 0\}$ is a Wiener process, M_t is the maximum value attained by W over the time interval $[0,t]$, Θ_t is the time the maximum value is attained, L_t is the local time of W at level zero and time t, and T_t is the last time W is zero in the time interval $[0,t]$. A straightforward proof, based on "Tanaka's formula", establishes this result by deriving an almost sure version of the equivalence.

AMS 1980 *subject classification.* Primary 60G17 Secondary 60H05

Keywords and phrases. Wiener process, Lévy's equivalence, Tanaka's formula

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As described by Knight (1981), "Lévy's equivalence" refers to the equality in distribution of the bivariate stochastic processes $\{(M_t - W_t, M_t), t \geq 0\}$ and $\{(|W_t|, L_t) t \geq 0\}$. Here, $W = \{W_t, t \geq 0\}$ is a (standard) Wiener process, $M_t = \max_{0 \leq s \leq t} W_s$, and L_t is the local time of W at level zero and time t. In a recent paper [3], the author presents an elementary derivation of a discrete analogue of this result, for a symmetric simple random walk, which he then uses to derive Lévy's equivalence. The objective here is to point out that there is a trivariate version of Lévy's equivalence, which states that the processes $\{(M_t - W_t, M_t, \Theta_t), t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions, where $\Theta_t \in [0,t]$ is the time at which the maximum M_t is attained, and T_t is the last zero of W in the time interval [0,t]. Additional keywords: Stochastic processes; distribution functions; Wiener process.

The proof depends on Tanaka's formula (cf. McKean (1969), page 68), which says

$$L_t = |W_t| + \tilde{W}_t \qquad , \; t \geq 0,$$

where $\tilde{W} = \{\tilde{W}_t, t \geq 0\}$ is a new Wiener process defined by

$$\tilde{W}_t = \int_0^t h(W_s) dW_s \qquad , \; t \geq 0,$$

with $h(\cdot) = -\text{sign}(\cdot)$ (cf. McKean (1969), page 29). Observe that

$$\tilde{W}_s - \tilde{W}_t = |W_t| - |W_s| - (L_t - L_s) \leq |W_t|, \; s \in [0,t].$$

The inequality is an equality if and only if $s = T_t$. Thus

The inequality is an equality if and only if $s = T_t$. Thus

$$(1) \qquad (\tilde{M}_t - \tilde{W}_t,\ \tilde{M}_t,\ \tilde{\Theta}_t) = (|W_t|,\ L_t,\ T_t) \qquad ,\ t \geq 0,$$

where $\tilde{M}_t = \max_{0 \leq s \leq t} \tilde{W}_s$, and $\tilde{\Theta}_t \in [0,t]$ is the time of the maximum. It should be emphasized that (1) is an almost sure identity in t for two trivariate stochastic processes. Consequently, $\{(M_t - W_t, M_t, \Theta_t), t \geq 0\}$ and $\{(|W_t|, L_t, T_t), t \geq 0\}$ have the same distributions as asserted.

REFERENCES

[1] Knight, F. (1981), Essentials of Browian Motion and Diffusion, Mathematical Surveys No. 18 (American Math. Soc., Providence).

[2] McKean, H.P. (1969), Stochastic Integrals, (Academic Press, New York).

[3] Simons, G. (1983), A discrete analogue and elementary derivation of 'Lévy's equivalence' for Browian motion, Statistics and Probability Letters 1, 203-206.

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